

Chapter 3: Gate Level Minimization

Solutions of Problems: [1, 4, 9, 13]

Problem 3-1

Simplify the following Boolean Functions, using three variable maps:

(a) $F(x,y,z) = \Sigma(0,2,6,7)$

	YZ			
X	00	01	11	10
0	1	0	0	1
1	0	0	1	1

$$F = XY + X'Z'$$

(b) $F(A,B,C) = \Sigma(0,2,3,4,6)$

	BC			
A	00	01	11	10
0	1	0	1	1
1	1	0	0	1

$$F = A'B + C'$$

(c) $F(a,b,c) = \Sigma(0,1,2,3,7)$

	bc			
a	00	01	11	10
0	1	1	1	1
1	0	0	1	0

$$F = a' + bc$$

(d) $F(x,y,z) = \Sigma(3,5,6,7)$

	YZ			
X	00	01	11	10
0	0	0	1	0
1	0	1	1	1

$F = XY + XZ + YZ$

Problem 3-4

Simplify the following Boolean Functions.

(a) $F(X,Y,Z) = \Sigma(2,3,6,7)$

	YZ			
X	00	01	11	10
0	0	0	1	1
1	0	0	1	1

$F = Y$

(b) $F(A,B,C,D) = \Sigma(4,6,7,15)$

	CD			
AB	00	01	11	10
00	0	0	0	0
01	1	0	1	1
11	0	0	1	0
10	0	0	0	0

$F = BCD + A'BD'$

(c) $F(A,B,C,D) = \Sigma(3,7,11,13,14,15)$

	CD			
AB	00	01	11	10
00	0	0	1	0
01	0	0	1	0
11	0	1	1	1
10	0	0	1	0

$F = CD + ABD + ABC$

(d) $F(W,X,Y,Z) = \Sigma(2,3,12,13,14,15)$

	YZ			
WX	00	01	11	10
00	0	0	1	1
01	0	0	0	0
11	1	1	1	1
10	0	0	0	0

$F = WX + W'X'Y$

Problem: 3-9

Find all the prime implicants for the following Boolean functions and determine which are essential:

- a) $F(w, x, y, z) = \sum(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$
b) $F(A, B, C, D) = \sum(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$
c) $F(A, B, C, D) = \sum(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

Solution:

Note: Red boxes represent the essential prime implicants.

- a) $F(w, x, y, z) = \sum(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$

		00	01	11	10	
		y				
00		1	0	0	1	
01		1	1	1	1	x
11		0	1	1	0	
10		1	0	0	1	
	w	z				

Essential: xz and $x'z'$; Non essential: $w'x$ and $w'z'$

$$F = xz + x'z' + w'z'$$

$$F = xz + x'z' + w'x$$

- b) $F(A, B, C, D) = \sum(0, 2, 3, 5, 7, 8, 10, 11, 14, 15)$

		00	01	11	10	
		c				
00		1	0	1	1	
01		0	1	1	0	B
11		0	0	1	1	
10		1	0	1	1	
	A	D				

Essential prime implicants: AC , $B'D'$ and $A'BD$

Prime implicants: CD and $B'C$

$$F = AC + B'D' + A'BD + CD$$

$$F = AC + B'D' + A'BD + B'C$$

c) $F(A, B, C, D) = \sum(1, 3, 4, 5, 10, 11, 12, 13, 14, 15)$

		C			
		00	01	11	10
A	00	0	1	1	0
	01	1	1	0	0
	11	1	1	1	1
	10	0	0	1	1
		D			

Diagram illustrating the Karnaugh map for the function $F(A, B, C, D)$. The map is a 4x4 grid with rows labeled A (00, 01, 11, 10) and columns labeled C (00, 01, 11, 10). The values in the cells are: (00,00)=0, (01,00)=1, (11,00)=1, (10,00)=0; (00,01)=1, (01,01)=1, (11,01)=0, (10,01)=0; (00,11)=1, (01,11)=1, (11,11)=1, (10,11)=1; (00,10)=0, (01,10)=0, (11,10)=1, (10,10)=1. Red boxes highlight the prime implicants: a 2x2 square covering (01,00), (11,00), (01,01), (11,01) labeled B; a 2x2 square covering (00,01), (01,01), (00,11), (01,11) labeled A; and a 2x2 square covering (11,01), (10,01), (11,11), (10,11) labeled D.

Essential prime implicants: BC' , AC and $A'B'D$

Prime implicants: AB (Please note that this prime implicant should **NOT** be included in your expression cause all the 1s are covered by the essential prime implicants)

$$F = AC + BC' + A'B'D$$

Problem: 3-13

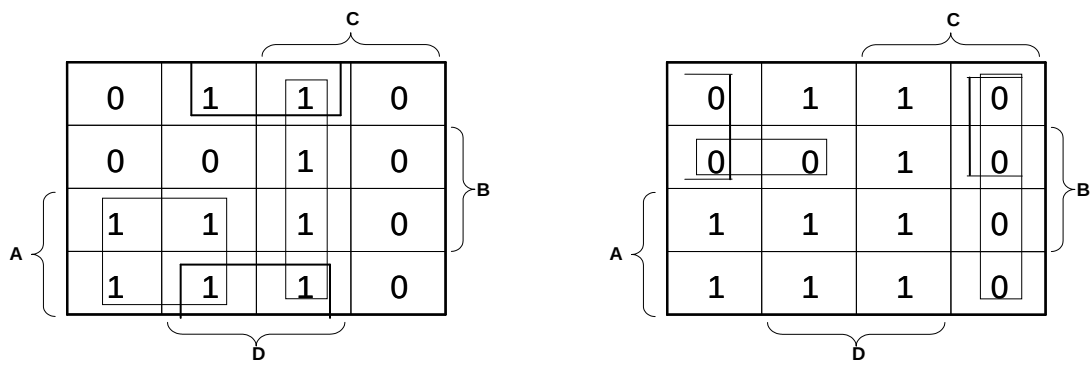
Simplify the following expressions in (1) sum of products and (2) products of sums:

- a) $x'z' + y'z' + yz' + xy$
 b) $AC' + B'D + A'CD + ABCD$
 c) $(A' + B' + D')(A + B' + C')(A' + B + D')(B + C' + D')$

Solution:

- a) $x'z' + y'z' + yz' + xy$
 SOP = $z' + xy$ POS = $(z' + x)(z' + y)$
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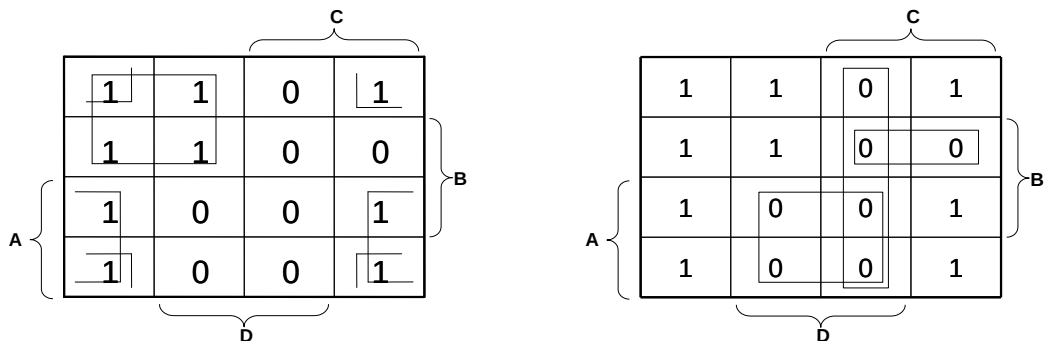
- b) $AC' + B'D + A'CD + ABCD$



SOP = $AC' + CD + B'D$

$F' = CD' + A'D' + A'BC'$
 POS = $(C' + D)(A + D)(A + B' + C)$

- c) $F = (A' + B' + D')(A + B' + C')(A' + B + D')(B + C' + D')$
 $F' = ABD + A'BC + AB'D + B'CD$



SOP = $A'C' + B'D' + AD'$

$F' = AD + CD + A'BC$
 POS = $(A' + D')(C' + D')(A + B' + C')$
